

REVISITING R. B. FULLER'S S & E MODULES

David B Koski
141 W 50th St
Minneapolis, MN, 55419, USA
E-mail:dbkoski@bitstream.net

Abstract

R Buckminster Fuller's Synergetic Geometry is known primarily for promoting the Tetrahedron as the standard unit of volume and the dissection of his Concentric Hierarchy of Polyhedra into A & B modules. The icosahedron in Fuller's Synergetics is intimately related to the cuboctahedron which Fuller named the Vector Equilibrium (VE). The relationship is demonstrated in Fuller's Jitterbug transformation from one form to the other. Buckminster Fuller realized the volumetric ratio between the VE and the icosahedron is incommensurable. He introduced three modules that are fully imbued within five-fold symmetry, the S, E and T modules. The T module is equivalent in volume to the A & B modules, the S & E are not. This paper will review the S, E and T modules with the golden ratio or phi and demonstrate a significant volumetric relationship between the VE and the Icosahedron. All of the following volumes are given in tetrahedral units, rounded at six decimal places. The conversion factor for cubic units is $(2\sqrt{2})/3 = .942809$. $\Phi = (1+\sqrt{5})/2$ or $1.618034 = \phi^1$

T & E Modules

The T-module is 1/120th of a rhombic triacontahedron, with the tetrahedral unit volume = 5. It has a radius of minutely close to being equal to 1, at $.999483 = ({}^3\sqrt{(2/3)(\phi^1)})/\sqrt{2}$. The radius described is from the origin to the center of the rhombic triacontahedron face. The rhombic triacontahedron that does have a radius of exactly 1, is dubbed the 5+ RT, having a volume of 5.007758. 1/120th of the 5+ rhombic triacontahedron is the E module. Fuller goes to great lengths in his Synergetics 2, to explain the relationship between the T and E modules. T module volume is $1/24 = .041666$, E module volume is $(\sqrt{2}/8)\phi^{-3} = .041731$

Phi Scaling

By scaling the edges of the E module, larger or smaller, by increments of ϕ^1 we increase or decrease the volume by phi to the third power. The notation used describes the various sizes of the E module as they are scaled by ϕ^1 and their volumes are greatened or lessened by ϕ^3 . Note the lower case "e" is used for the ϕ^{-3} increments. "E0" = "e0", but "e" nor the "0" are utilized.

E module denotations

$$e6 = ((\sqrt{2}/8)\phi^{-9}) = .002325$$

$$e3 = ((\sqrt{2}/8)\phi^{-6}) = .009851$$

$$E = ((\sqrt{2}/8)\phi^{-3}) = .041731$$

$$E3 = ((\sqrt{2}/8)\phi^0) = .176766$$

$$E6 = ((\sqrt{2}/8)\phi^3) = .748838$$

The E module can be made of lesser scaled modules with the general volumetric relationship: $1E3 = 4E + 1e3 = 17e3 + 4e6$ and so on.

A rhombic triacontahedron with a radius of ϕ^1 , is dubbed the Super RT. The long diagonal of the rhombic face = 2, which is R.B.Fuller's edge for the tetrahedron, octahedron, cuboctahedron or VE, and the resultant icosahedron from the Jitterbug transformation. The volume of the Super RT is $15\sqrt{2}$ or $21.213203 = 120E^3 = 480E + 120e^3$. The icosahedron with an edge = 2, inscribes within the Super RT. It has a volume of $5(\sqrt{2})\phi^2 = 18.52295$. It has an exact E module volume of $100E^3 + 20E = 420E + 100e^3$.

S Module

The S module was most likely named because of the "skewing" of an icosahedron inside of an octahedron. Where eight of the triangular faces of the icosahedron are co-planar with the octahedron it is inscribed within. The difference in volume is : (Octahedron= 4) - (icosahedron= 2.917960) = 24 S modules. The The volume left over can be sectioned into 24 equal modules, 12 left and 12 right handed tetrahedra. Fuller's choice of using a volume 4 octahedron must have been primarily intuitive, but forcibly positioned to get the S module as close as possible to the volume of an A&B module. The volume of the S module is $(\phi^{-5})/2 = .045084$.

S & E Modules

The VE or cuboctahedron with an edge = 2, has a volume of 20 and it Jitterbugs to the icosahedron with a volume of 18.512295. The ratio of their volumes is 1.080363 or $2(\sqrt{2})\phi^{-2}$. We can express the volume of the VE using phi-scaling and the same style of denotation:

$$VE = 100S^3 + 20S = 420S + 100s^3$$

The ratio of is then written $(420S + 100s^3)/(420E + 100e^3) = 1.080363$.

The concentric hierarchy can be described in terms of phi-scaled S modules:

Tetrahedron: $21S+5s^3 = 5S^3+1S = 1S^6+1S^3$

Cube: $72S+15s^3 = 15S^3+3S = 3S^6+3S^3$

Octahedron: $84S+20s^3 = 20S^3+4S = 4S^6+4S^3$

Rh Triac: $105S+25s^3 = 25s^3+5S = 5S^6+5S^3$ (volume = 5)

Rh Dodec: $126S+30s^3 = 30S^3+6S = 6S^6+6S^3$

Conclusion

Two seemingly "dead end" modules, the Synergetics S & E, have a very intimate volumetric relationship with the Jitterbug transformation of the VE and Icosahedron. We find that the T module with its equivalency to the A&B modules is the "gateway" module for five-fold symmetric forms. The closely related E module, having the exact angles and form to the T module, expresses the volumes of the five-fold symmetric forms and the S module brings the five-fold back to the four-fold symmetry utilizing phi-scaling.

T & E MODULES' LENGTHS

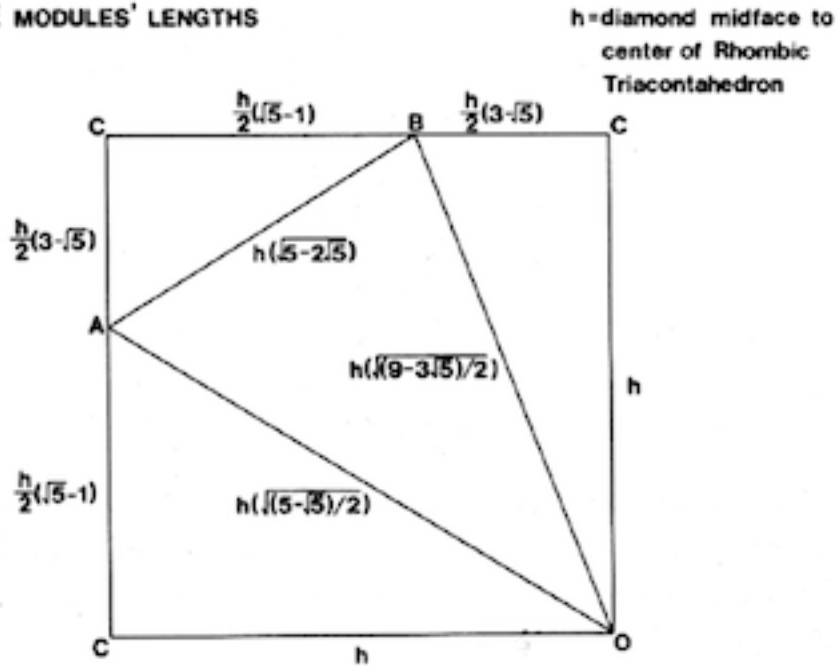


Fig. 986.411A T and E Quanta Modules: Edge Lengths: This plane net for the T Quanta Module and the E Quanta Module shows their edge lengths as ratioed to the octa edge. Octa edge = tetra edge = unity. [4]

S MODULE LENGTHS

a=edge of octahedron

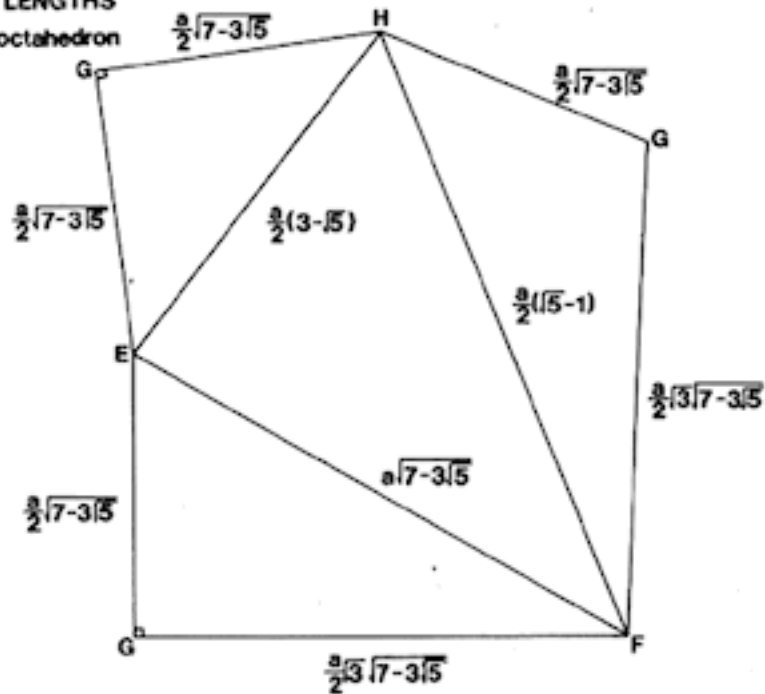


Fig. 988.13A S Quanta Module Edge Lengths: This plane net for the S Quanta Module shows the edge lengths ratioed to the unit octa edge (octa edge = tetra edge.) [5]

References

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- [5] <http://www.rwgrayprojects.com/synergetics/s09/figs/f8813a.html>